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# Anisotropic Elastic Constants Calculation of Stainless Steel Cladded Layers of Pressure Vessel Steel Plate

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**Keywords:** pressure vessel, stainless steel cladded layer, anisotropy, elastic constants

**Abstract.** Cladding stainless steel layer on the inner surface of ferrite pressure vessel is a common method to improve the corrosion resistance and save the economic cost. However, the movement of heat source and temperature gradient in the process of cladded welding will lead to the anisotropy of cladded layer material. When measuring the residual stress of pressure vessel steel plate with stainless steel cladded layers by contour method, it is necessary to know the elastic mechanical properties of stainless steel cladded layers accurately. The assumption of transversely isotropy was employed, and the relationship between the material compliance matrix and the elastic modulus of transversely isotropic material was utilized. Based on the elastic modulus of each cladded layer and the whole steel plate from the longitudinal direction ( $0^\circ$ ) until the transverse direction ( $90^\circ$ ) measured by the experiment, the independent constants  $S11$ ,  $S13$ ,  $S33$  and  $S44$  in the compliance matrix of each cladded layer and the whole steel plate were obtained by regression analysis method. Furthermore, by using the relationship between the independent constants of the stiffness matrix of the transversely isotropic material and the single crystal material, the independent constants  $S12$  in the compliance matrix of each stainless steel cladded layer and the whole steel plate were obtained. And then the independent constants of the stiffness matrix of each cladded layer and the whole steel plate were acquired. Hence, a method for calculating the anisotropic elastic constants of the stainless steel cladded layer and the whole steel plate was proposed. The results will provide material data support for measuring residual stress of pressure vessel steel plate with stainless steel cladded layers by contour method.

## Introduction

Pressure vessels are the most important and widely used component in power, petroleum and chemical industries. Using stainless steel cladded layer on the inner surface of ferrite pressure vessel is a common method to improve the corrosion resistance and save the economic cost. However, the cladded welding is a complex process. During the process of cladded welding, the movement of heat source and temperature gradient will lead to the anisotropy of cladded layer material [1]. For welded structures, the welding residual stress has an important effect on their service performance. So, it is essential to know the welding residual stress distribution in the welded structures. The same is true for the stainless steel cladded layer of pressure vessel steel plate.

The contour method is a burgeoning method for welding residual stress measurement [2]. Many cases show that the contour method is an effective and precise method for different welding methods [3]. Implementing the contour method includes four steps: cutting the part, measuring the surface, processing the data and evaluating the residual stress. In the last step, the finite element model needs to be constructed, applying the profile measured on the cutting surface, to capture the residual stress. In order to obtain the precise residual stress, the input of the accurate elastic mechanical property is

essential in the finite element analysis (FEA). When measuring the residual stress of the stainless steel cladded layer of pressure vessel steel plate by the contour method, the accurate elastic mechanical properties of the stainless steel cladded layer of pressure vessel steel plate are needed.

In this paper, based on the elastic modulus of each cladded layer and the whole steel plate from the longitudinal direction ( $0^\circ$ ) until the transverse direction ( $90^\circ$ ) measured by the experiment, the authors proposed a method to calculate the independent elastic constants in the compliance matrix and stiffness matrix of each cladded layer and the whole steel plate, which can provide the material data support for measuring residual stress of pressure vessel steel plate with stainless steel cladded layers by contour method.

## Experimental

The experiments of welding pressure vessel steel plate with stainless steel cladded layers were conducted by Rebelo Kornmeier [1]. The samples were made from an EN 10028-3 (P355 NH) carbon steel plate of 20 mm thickness. One of the faces, the carbon steel plate was cladded by SAW, using an austenitic filler metal of 4 mm in diameter. The first welding layer was made with an EN 12072-S 23 12 2 L (AISI 309L) electrode in order to minimize metallurgical problems. The second and the third layers were made with an EN 12072-S 19 12 3 L (AISI 316L) electrode. Among different cladding procedures the automated submerged arc welding (SAW) was generally employed due to its high quality and reliability. This procedure leads to the production of welded specimens with a final thickness of 27.5 mm. The optical microstructure of the clad is shown in Figure 1 (a). One of the investigated samples was as welded (AW sample) and the other two after weld and subsequent two different heat treatments, one at  $620^\circ\text{C}$  for a holding time of 1 hour (HT620 sample) and the other at  $540^\circ\text{C}$  for a holding period of 10 hours (HT540 sample).

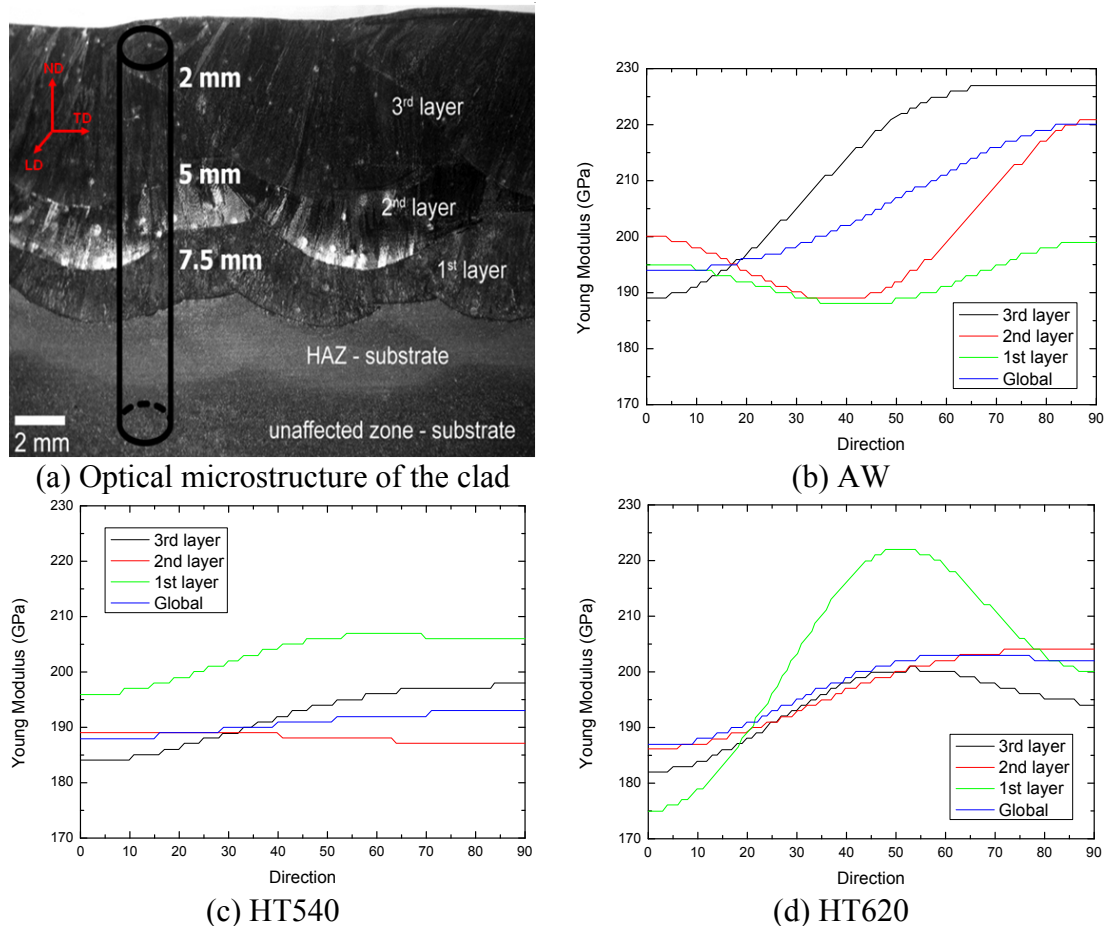


Fig. 1 Optical microstructure of the clad (a) and local and bulk Young modulus from LD ( $0^\circ$ ) to TD ( $90^\circ$ ) for (b) AW, (c) HT540 and (d) HT620 sample respectively

The three main directions, longitudinal (LD), transverse (TD) and normal (ND), of the welding plates were represented in Figure 1 (a). After texture measurements and orientation distribution functions (ODFs) calculation using the measured complete pole figures with harmonic series expansion method, with the calculated C-coefficients, the tri-axial Young modulus were calculated using Cub\_PHY program based on a cluster model [1]. The Young modulus (local and bulk) values of all samples were plotted in Figure 1 (b~d) for the different directions and the different layers. For each sample the Young modulus were plotted from the longitudinal direction (0°) until the transverse direction (90°).

### Elastic Constants Calculation

For anisotropic materials, there are:

$$\bar{\sigma}_i = \sum C_{ij} \cdot \varepsilon_j \quad (1)$$

$$\bar{\varepsilon}_i = \sum S_{ij} \cdot \sigma_j \quad (2)$$

Where,  $C_{ij}$  is stiffness matrix, and  $S_{ij}$  is compliance matrix. The stiffness matrix  $C_{ij}$  can be calculated from the compliance matrix  $S_{ij}$  and vice versa.

For austenitic weld metal with a columnar grain structure, a common assumption is that the material is transversely isotropic. The experiment exhibits that the type of grain structure about the austenitic weld is columnar grain structure [4]. So, each layer of the stainless steel caldded layer can be approximately simplified as the transversely isotropic material. The stiffness matrix and compliance matrix of the transversely isotropic material have five independent constants,  $C11$ ,  $C12$ ,  $C13$ ,  $C33$  and  $C44$ ,  $S11$ ,  $S12$ ,  $S13$ ,  $S33$  and  $S44$ , respectively. The Equations (1) and (2) can be written as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & (C_{11} - C_{12})/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & & & \\ S_{12} & S_{11} & S_{13} & & & \\ S_{13} & S_{13} & S_{33} & & & \\ & & & S_{44} & & \\ & & & & S_{44} & \\ & & & & & 2(S_{11} - S_{12}) \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} \quad (4)$$

For transversely isotropic material, the relations between the direction-dependent elastic modulus  $E$  and shear modulus  $G$  and the five independent constants in compliance matrix are given by Equations (5) and (6) [5]:

$$E(\theta) = [S_{11} \sin^4 \theta + S_{33} \cos^4 \theta + (2S_{13} + S_{44}) \sin^2 \theta \cos^2 \theta]^{-1} \quad (5)$$

$$G(\theta) = [S_{44} + (S_{11} - S_{12} - (S_{44}/2)) \sin^2 \theta + 2(S_{11} + S_{33} - 2S_{13} - S_{44}) \sin^2 \theta \cos^2 \theta]^{-1} \quad (6)$$

Where,  $\theta$  is the angle between the specimen axis and the fibre axis.

Based on the data of local and bulk Young modulus in Figure 1 (b~d), the regression analysis method can be used to derive the  $S11$ ,  $S13$ ,  $S33$  and  $S44$  of every stainless steel caldded layer and bulk. The regression equation used are shown as Equation (7):

$$\frac{1}{E(\theta)} = S_{11} \sin^4 \theta + S_{33} \cos^4 \theta + (2S_{13} + S_{44}) \sin^2 \theta \cos^2 \theta \quad (7)$$

The regression results of  $S_{11}$ ,  $S_{13}$ ,  $S_{33}$  and  $S_{44}$  of every stainless steel cladded layer and bulk are listed in Table 1. And the regression analysis curves are shown in Figure 2~4.

Table 1 Regression results of  $S_{11}$ ,  $S_{13}$ ,  $S_{33}$  and  $S_{44}$  (1/GPa)

Sample		3 <sup>rd</sup> layer	2 <sup>nd</sup> layer	1 <sup>st</sup> layer	Global
AW	$S_{11}$	0.00441	0.00452	0.00502	0.00454
	$S_{13}$	0.00241	0.00314	0.00303	0.00272
	$S_{33}$	0.00529	0.00500	0.00512	0.00516
	$S_{44}$	0.00383	0.00528	0.00506	0.00444
HT540	$S_{11}$	0.00506	0.00535	0.00486	0.00518
	$S_{13}$	0.00281	0.00290	0.00263	0.00286
	$S_{33}$	0.00544	0.00529	0.00510	0.00532
	$S_{44}$	0.00461	0.00480	0.00425	0.00472
HT620	$S_{11}$	0.00515	0.00490	0.00500	0.00495
	$S_{13}$	0.00260	0.00273	0.00211	0.00266
	$S_{33}$	0.00549	0.00537	0.00571	0.00536
	$S_{44}$	0.00420	0.00445	0.00322	0.00432

Correlation coefficient ( $R$ ) is a commonly used statistical parameter and provides information of relationship between the predicted results by regression formulas and the experimental results. It can be mathematically expressed as:

$$R = \frac{\sum_{i=1}^{i=N} ((E)_{EXP}^i - (\bar{E})_{EXP}) ((E)_{REG}^i - (\bar{E})_{REG})}{\sqrt{\sum_{i=1}^{i=N} ((E)_{EXP}^i - (\bar{E})_{EXP})^2 \sum_{i=1}^{i=N} ((E)_{REG}^i - (\bar{E})_{REG})^2}} \quad (8)$$

Where  $(E)_{EXP}$  and  $(E)_{REG}$  are the Young modulus obtained by experiments and predicted by regression formulas respectively;  $(\bar{E})_{EXP}$  and  $(\bar{E})_{REG}$  are the mean values of  $(E)_{EXP}$  and  $(E)_{REG}$ , respectively;  $N$  is the number of the statistical samples.

The correlation coefficients of regression formulas are listed in Table 2. From this table, the minimum correlation coefficient was 0.9129. So, the results of regression formulas were effective and believable.

Table 2 Correlation coefficients of regression results

Sample	3 <sup>rd</sup> layer	2 <sup>nd</sup> layer	1 <sup>st</sup> layer	Global
AW	0.9992	0.9984	0.9832	0.9977
HT540	0.9906	0.9129	0.9863	0.9510
HT620	0.9960	0.9953	0.9994	0.9941

Because the shear modulus of the stainless steel cladded layer is unknown, the  $S_{12}$  could not be obtained for the time being. Here, assuming  $S_{12}=x$ , the compliance matrix  $S_{ij}$  can be obtained. Then, inversing the compliance matrix  $S_{ij}$ , the stiffness matrix  $C_{ij}$  can be obtained. It is important to note that the compliance matrix  $S_{ij}$  and the stiffness matrix  $C_{ij}$  contain unknown variable  $x$ .

The stiffness matrix of single crystal material contains three independent elastic constants,  $c_{11}$ ,  $c_{12}$  and  $c_{44}$ . The relationship between three independent constants of single crystal material and five independent constants of transversely isotropic material is as follows [6]:

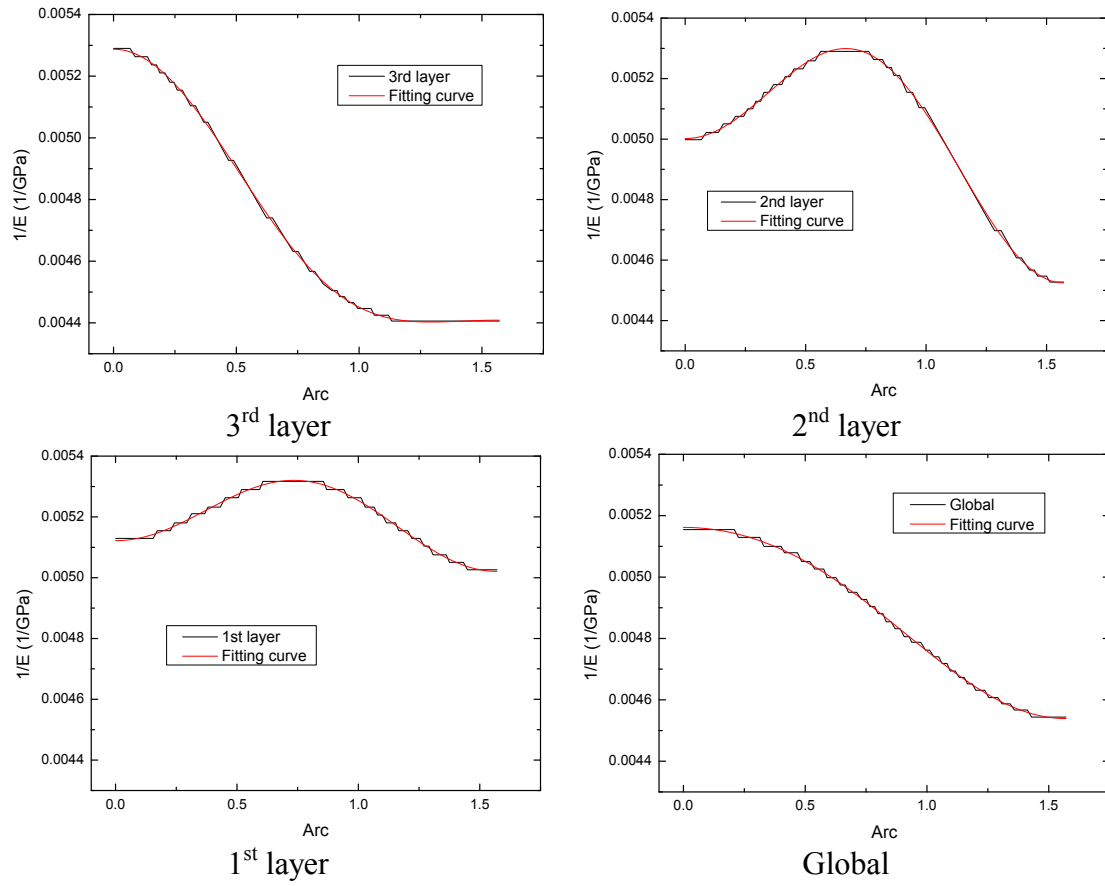


Fig. 2 Regression of local and bulk Young modulus for AW sample

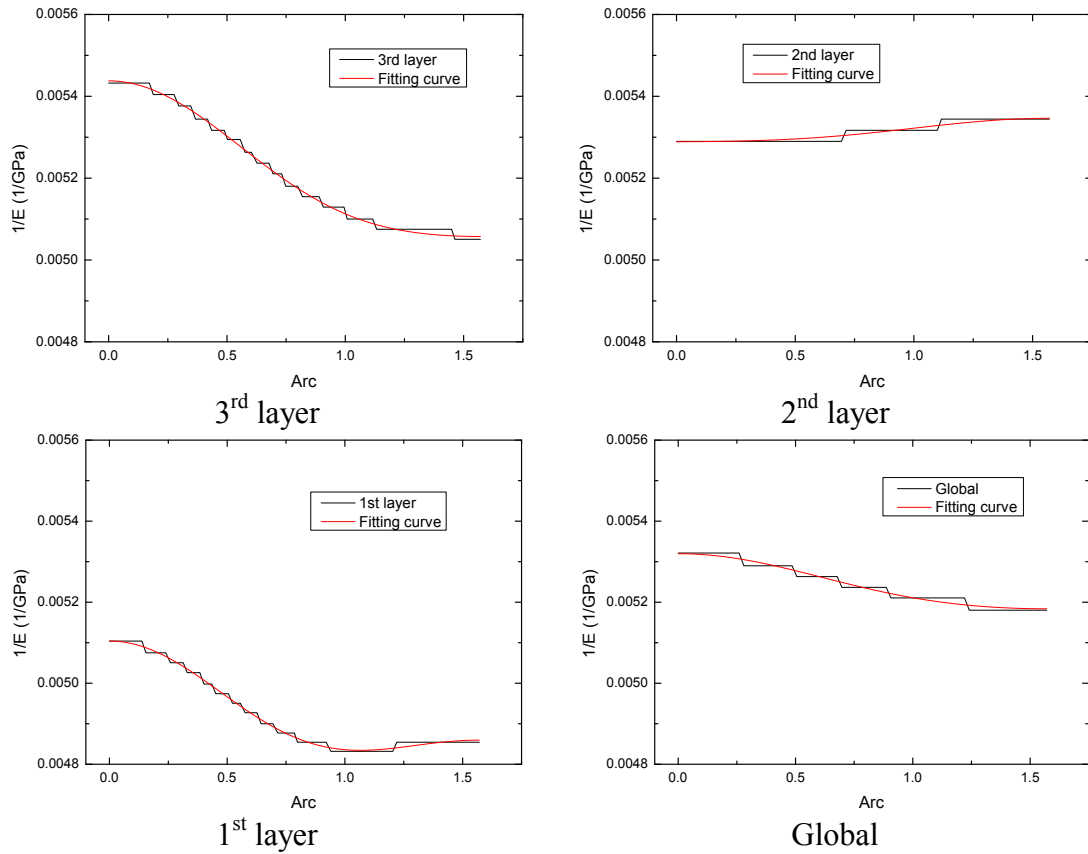


Fig. 3 Regression of local and bulk Young modulus for HT540 sample

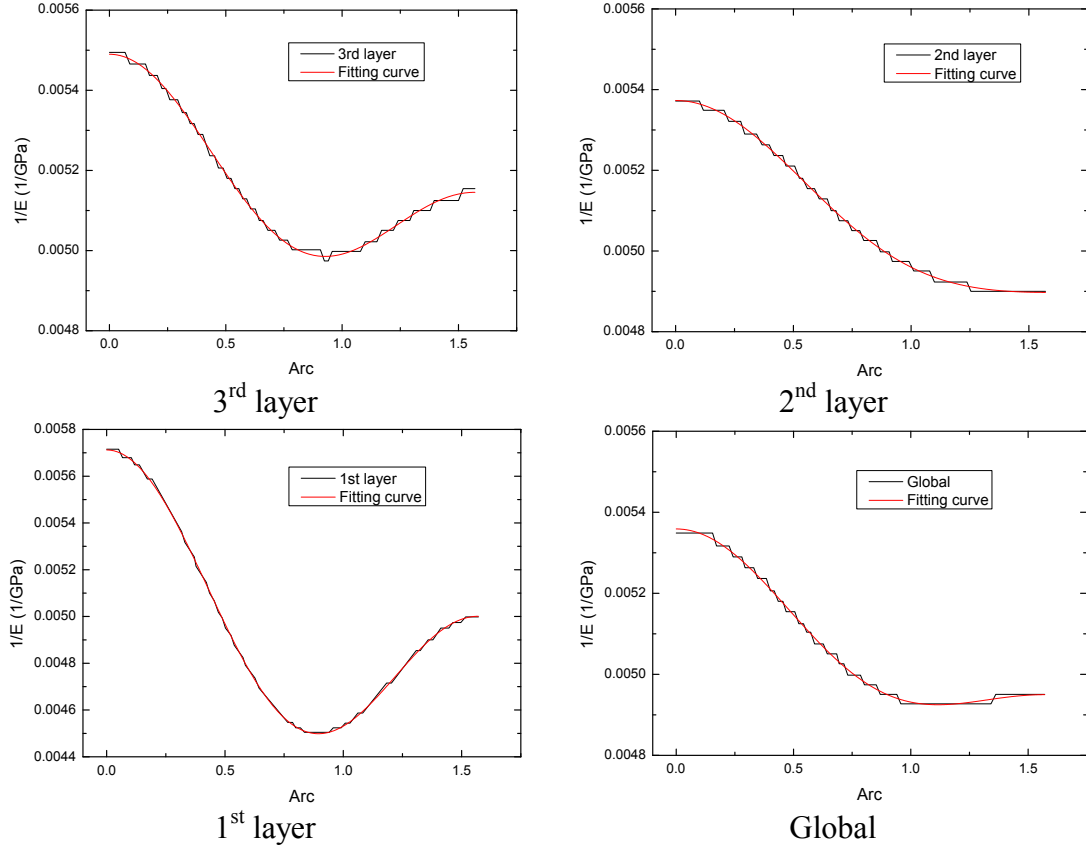


Fig. 4 Regression of local and bulk Young modulus for HT620 sample

$$C_{11} = \frac{1}{4}(3c_{11} + c_{12} + 2c_{44}) \quad (9)$$

$$C_{12} = \frac{1}{4}(c_{11} + 3c_{12} - 2c_{44}) \quad (10)$$

$$C_{33} = c_{11} \quad (11)$$

$$C_{13} = c_{12} \quad (12)$$

$$C_{44} = c_{44} \quad (13)$$

Substituting the Equations (11) ~ (13) into Equation (10), the Equation (14) can be obtained:

$$C_{12} = \frac{1}{4}(c_{11} + 3c_{12} - 2c_{44}) = \frac{1}{4}(C_{33} + 3C_{13} - 2C_{44}) \quad (14)$$

Because  $C_{13}$ ,  $C_{33}$  and  $C_{44}$  of transversely isotropic material are known, the  $x$  can be obtained according to Equation (14). Then,  $S_{12}$  and  $C_{12}$  of transversely isotropic material can be obtained, too. The calculation results of  $S_{12}$  of every stainless steel caldded layer and bulk are shown in Table 3. And the calculation results of elastic constants in stiffness matrix are shown in Table 4.

It should be noted that  $C_{12}$  and  $C_{13}$  are negative. This is because that among the five independent constants of the stiffness matrix  $C_{ij}$  ( $C_{11}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{33}$  and  $C_{44}$ ),  $C_{11}$ ,  $C_{33}$  and  $C_{44}$  (when  $i=j$ ) are related to the principal stress component, while  $C_{12}$  and  $C_{13}$  (when  $i \neq j$ ) are related to the shear stress component, which are direction dependent, and the negative values mean the opposite direction.

Table 3 Calculation results of  $S_{12}$  (1/GPa)

Sample		3 <sup>rd</sup> layer	2 <sup>nd</sup> layer	1 <sup>st</sup> layer	Global
AW	$S_{12}$	0.0024	0.0027	0.0028	0.0025
HT540	$S_{12}$	0.0027	0.0029	0.0026	0.0028
HT620	$S_{12}$	0.0028	0.0027	0.0028	0.0027



Table 4 Calculation results of elastic constants in stiffness matrix (GPa)

Sample		3 <sup>rd</sup> layer	2 <sup>nd</sup> layer	1 <sup>st</sup> layer	Global
AW	<i>C11</i>	353.65	430.28	346.62	365.13
	<i>C12</i>	-135.25	-127.58	-112.82	-125.59
	<i>C13</i>	-99.50	-190.10	-138.36	-126.27
	<i>C33</i>	279.69	438.76	359.08	326.92
	<i>C44</i>	261.10	189.39	197.63	225.23
HT540	<i>C11</i>	318.18	304.34	330.17	314.03
	<i>C12</i>	-114.29	-108.10	-121.57	-111.37
	<i>C13</i>	-105.32	-107.58	-107.57	-108.95
	<i>C33</i>	292.62	306.99	307.03	305.11
	<i>C44</i>	216.92	208.33	235.29	211.86
HT620	<i>C11</i>	301.44	327.60	305.77	319.83
	<i>C12</i>	-118.08	-118.51	-145.06	-119.10
	<i>C13</i>	-86.84	-106.30	-59.39	-99.62
	<i>C33</i>	264.40	294.30	219.02	285.44
	<i>C44</i>	238.10	224.72	310.56	231.48

### Material Properties Needed in FEA

Finally, when measuring residual stress of pressure vessel steel plate with stainless steel cladded layers by contour method, during the finite element analysis, the material properties of each stainless steel cladded layer and the whole steel plate, which can be indicated as elastic material properties of the transversely isotropic material, can be applied by defining the elastic stiffness matrix of the orthotropic elasticity, as shown in Table 5.

Table 5 Material properties needed in FEA (GPa)

Sample		D1111	D2222	D3333	D1122	D1133	D2233	D1212	D1313	D2323
AW	3 <sup>rd</sup> layer	353.65	353.65	279.69	-135.25	-99.50	-99.50	261.10	261.10	244.45
	2 <sup>nd</sup> layer	430.28	430.28	438.76	-127.58	-190.10	-190.10	189.39	189.39	278.93
	1 <sup>st</sup> layer	346.62	346.62	359.08	-112.82	-138.36	-138.36	197.63	197.63	229.72
	Global	365.13	365.13	326.92	-125.59	-126.27	-126.27	225.23	225.23	245.36
HT540	3 <sup>rd</sup> layer	318.18	318.18	292.62	-114.29	-105.32	-105.32	216.92	216.92	216.23
	2 <sup>nd</sup> layer	304.34	304.34	306.99	-108.10	-107.58	-107.58	208.33	208.33	206.22
	1 <sup>st</sup> layer	330.17	330.17	307.03	-121.57	-107.57	-107.57	235.29	235.29	225.87
	Global	314.03	314.03	305.11	-111.37	-108.95	-108.95	211.86	211.86	212.70
HT620	3 <sup>rd</sup> layer	301.44	301.44	264.40	-118.08	-86.84	-86.84	238.10	238.10	209.76
	2 <sup>nd</sup> layer	327.60	327.60	294.30	-118.51	-106.30	-106.30	224.72	224.72	223.05
	1 <sup>st</sup> layer	305.77	305.77	219.02	-145.06	-59.39	-59.39	310.56	310.56	225.42
	Global	319.83	319.83	285.44	-119.10	-99.62	-99.62	231.48	231.48	219.46

### Conclusions

The anisotropy of pressure vessel steel plate with stainless steel cladded layers was considered in this paper. Because the type of grain structure about the cladding stainless steel layer shows columnar grain structure, the each layer of the stainless steel cladded layers can be approximately simplified as the transversely isotropic material.

Using the assumption of transversely isotropy, based on the elastic modulus of each cladded layer and the whole steel plate from the longitudinal direction (0°) until the transverse direction (90°) measured by the experiment, the independent constants *S11*, *S13*, *S33* and *S44* in the compliance matrix of each cladded layer and the whole steel plate were obtained by regression analysis method. Next, the independent constants *S12* in the compliance matrix of each stainless steel cladded layer and the whole steel plate were obtained by using the relationship between the independent constants of the stiffness matrix of the transversely isotropic material and the single crystal material.

Then, a method to calculate the independent elastic constants in the compliance matrix and stiffness matrix of each clad layer and the whole steel plate was proposed, which can provide the material data support for measuring residual stress of pressure vessel steel plate with stainless steel clad layers by contour method.

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